Fractional Derivatives In Equivalent Electrical Circuits, Evidence From The Biological Tissue Sample

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***Abstract* – This article describes a computer model structure for an equivalent electrical circuit of the homogeneous biological tissue fragment, through the example of the porcine muscle sample, as well as the method of its parameters alignment according to the real object characteristics based on the impedances amplitude-frequency dependences matching.**

To implement the set goal standard electronic components, condensers, induction coils, and limited resistors used in the equivalent electrical circuit were replaced with programmable current sources. This makes possible to provide flexible adjustment of the functional dependence for both, separate components, and the resulting impedance amplitude-frequency response of the biological tissue in general.

Adjustment of the programmable current sources is performed due to application of the direct formalized description of the required functional dependence.

Applied formulas interpret presentation of fractional integrals and derivatives in the form of Laplasian operator, and make it possible to change slope of the frequency dependence curve relative to the reference axes.

The measurement graphs, correcting constants and analytical relations determined during PC simulation are represented as the results obtained for samples of the porcine tissue and porcine liver.

***Keywords* – fractional integrals, equivalent electrical circuit, biological tissue, Laplasian operator.**

I. INTRODUCTION

THE issue of the biological tissue consideration as an electrical circuit element rises every time when it is necessary to analyze the process of the biological tissue and electric current interaction. On the one hand the thermal effect of the flowing current induces reversible and irreversible changes of the biological tissue parameters, and on the other hand, parameters of biological tissue effect on the current flowing terms.

In this case of the model practical application it is considered appropriate to use the 1D variant of the equivalent circuit. It can be done with the aim to optimize the simulation velocity, and in connection with the wide spread of the practice of applying experimental and test devices of electrosurgical aggregates. In fact, biological tissue can be analyzed by means, of its connection, for example, to electrodes of an electrical surgical machine, and these are just two electric contacts degrading all the variety of the tissue properties applicable to its equivalent two-terminal device.

Due to the relatively low frequency of alternate current applied in electrical surgery, approx. 1 MHz, I consider it possible to neglect the wave effects and examine the thermal effect on the active component of the equivalent circuit and its frequency dependence on the reactive component effect.

Let us examine the well-known simplified equivalent circuit, which represents a combination of two resistors and one condenser.

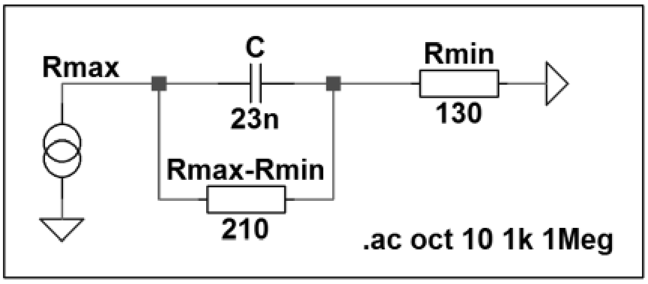

Fig. 1: Popular and the most distributed biological tissue equivalent circuit

We shall examine in more details the equivalent circuit components and their effect on developing the circuit frequency function schedule with the aim of the direct affection and alteration of the required parameters for equalizing graphs of the simple equivalent circuit simulation and results of measuring a biological tissue sample.

Thus, the single RCR structure impedance value and nature of its functional dependence is mainly determined by the condenser frequency dependence and thus, the first order derivative function of the voltage applied within the «Rmax» upper limit and «Rmin» lower limit.

$$Z_{RCR} = Rmin + \frac{R_C(Rmax - Rmin)}{R_C + (Rmax - Rmin)}$$

$$R_C \sim \frac{d}{dt}u$$

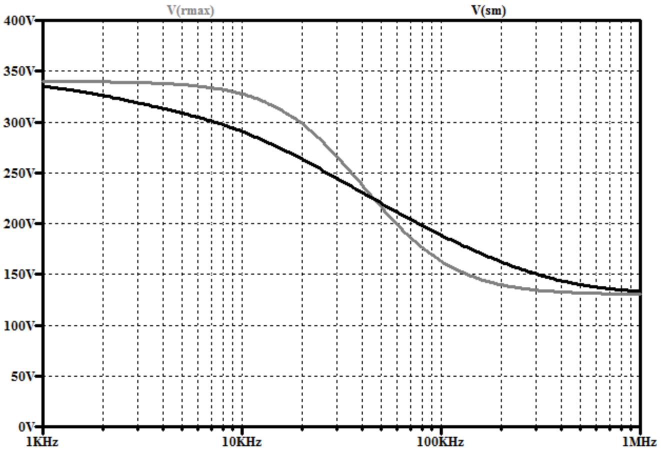

Fig. 2: Porcine muscle biological tissue impedance function of the simple equivalent circuit simulation result at the frequency range of 1 kHz to 1 MHz. The equivalent resistance in Ohm/V is marked along the Y-axis, and the measured voltage frequency in Hz is marked along the X-axis. The measured characteristics is represented in black, and the result of the simple equivalent circuit simulation is shown in gray

II. RESEARCH OBJECTIVE

It is obvious that for the arbitrary change of the slope characteristics for the integrated equivalent circuit it is necessary to find the way of affecting the frequency characteristics applied by a condenser. For this purpose we will replace a condenser in the suggested simple equivalent circuit with a controllable current source.

By definition, value of the current flowing via the condenser is proportional to the first-order derivative of the applied voltage.

$$i = a\frac{du}{dt}$$

For operating with derivatives we will use the Laplace's equation with the following modifications. Replace capacity value in this equation with the generalized coefficient, and denote it by the "k" letter. Let's introduce the power index "a" for the "s" parameter.

The transformed variant of the Laplace's equation acquires the form:

$$Laplace = k \cdot s^a ,$$

where $k = b \cdot \exp(c \cdot a)$,

where, in its turn, for the porcine tissue:

$b = 0{,}00402$
$c = -12.06$
$a \rightarrow$ derivative parameter

Next, compare simulation results at the power index step change.

Based on the preliminary analysis, it was obtained that the suggested process description maintains its accuracy for at least the range of the power index half-interval change. Changes with the 1/3 pitch displayed its applicability for achieving the set goal, i.e., the search of the frequency function for the biological tissue polarization and substituting integrated equivalent circuit. For simulating we used derivative values approximated to the following:

$$4/3, \quad 3/3, \quad 2/3, \quad 1/3$$

As well it should be noted that for the porcine liver the «*b*» adjusting factor reduces by approximately 10 times, and the «*c*» parameter displays insignificant changing that весьма allows to consider it to be a constant.

$$b = 0.419m \quad c = -12.2$$

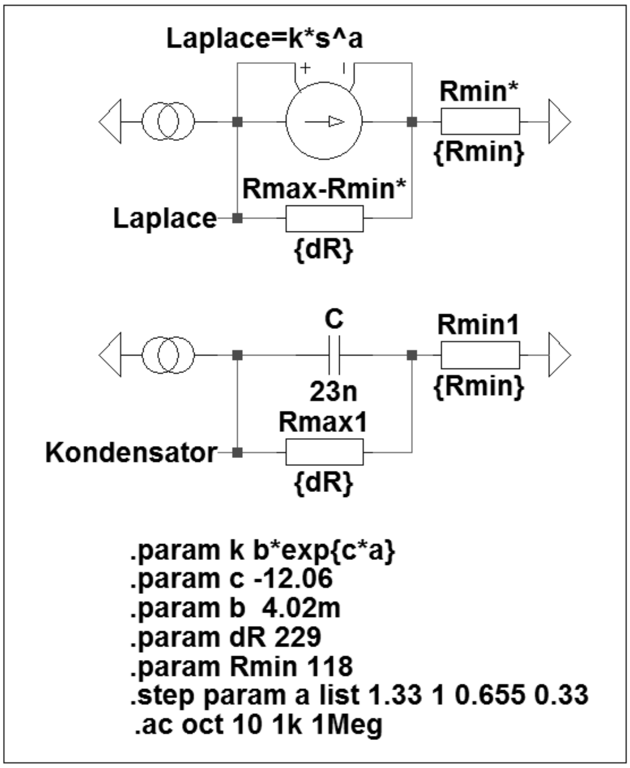

Fig. 3: Modified based on the Laplasian operator (on top) and the initial (below) variants of the simple equivalent circuit with a step-based changing derivative order for the porcine muscle tissue sample

One of the gray lines almost coincides with the black line, thus, to improve graph reading capacity gray lines were artificially shifted sideward by a slight change of the "b" parameter.

Gray – Variants of frequency characteristics for the circuit using Laplasian operator at various power index value

Black – Frequency characteristics for the circuit using a condenser.

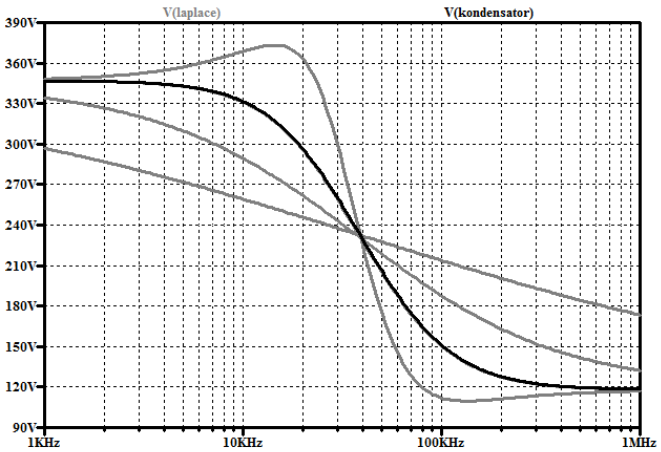

Fig. 4: Frequency dependence graphs for the Laplasian operator-based modification of (a number of gray graphs) and initial (black graph) simple equivalent circuit developed for simulating the porcine muscle tissue sample

Equivalent circuit with distributed parameters [1] displayed its high compliance accuracy of the received frequency characteristics with the biological tissue parameters; thus, we can use it as a primary comparison reference, which is especially suitable during joint simulation of the both circuits' variants, sequential search and optimization of adjusting factors.

To clearly understand multipurpose of the equivalent circuit structure and to see the content and difference of parameters in the course of simulating various samples of biological tissue, we will conduct their tests simultaneously, and represent result graphs gradually.

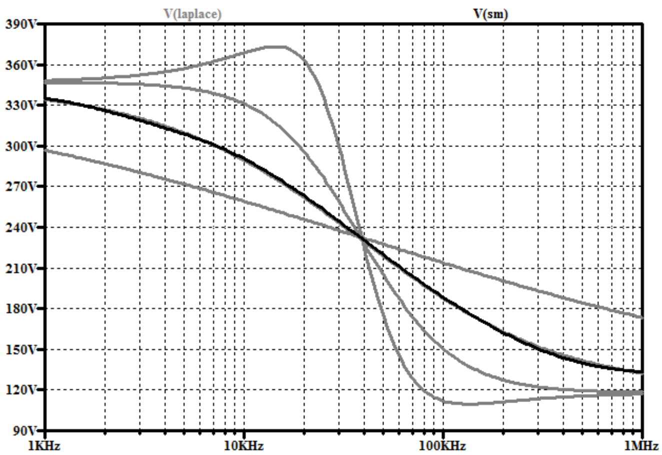

Fig. 5: Frequency dependence graphs for the Laplasian operator-based modification of (a number of gray graphs) and the semi-segment series RC equivalent circuit (black graph).

For measuring basic impedance characteristics of the real samples of biological tissue, porcine muscle tissue and porcine liver not more than two hours fresh, we used to some extend idealized samples.

These samples have uniform structure and are relatively homogeneous.

Probably it is another reason, why graphs of theoretical computer simulation results comply with the real objects graphs. Sure enough that only two types of the relatively homogeneous samples of biological tissue is not enough for generalization, however we can make a supposition that impedance value relative to the biological tissue with the homogeneous structure is determined by the fractional derivative function.

For the suggested samples of the porcine muscle tissue and porcine liver the derivative order is close to 2/3. In other words, impedance of the biological tissue fragment is proportional to the fractional derivative of the applied voltage

$$\mathrm{Z_{bio}} \sim \frac{\mathrm{d}^{2/3}}{\mathrm{dt}^{2/3}}\mathrm{u}$$

Laplasian operator significantly simplifies transfer to the fractional derivative. For this aim the following fractional power is used

$$\mathrm{Z_{bio}} \sim \mathcal{L}\,\{s^{2/3}\}$$

Biological tissues with mixed structure experience certain "blur" of functional dependence due to superposing parameters of one tissue to another.

In my opinion, the core goal on searching the way of forming the functional dependence for the simple equivalent circuit was solved. The only task remained is to enter limit conditions into the formalized equation in the form of resistors from the equivalent circuit

$$Laplace = 1/(Rmin + 1/(1/(Rmax - Rmin) + k \cdot s^{a}))$$

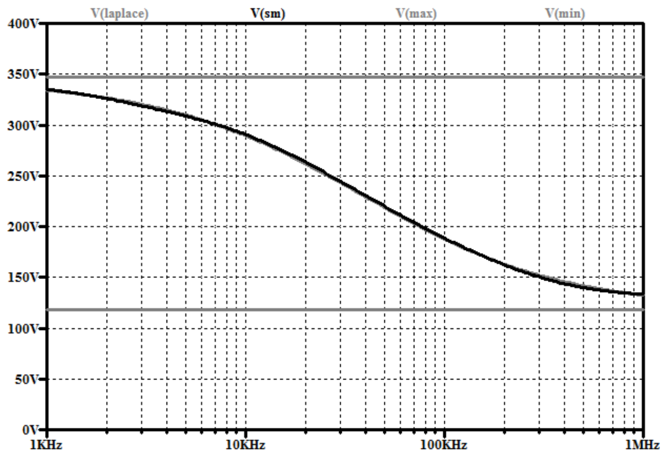

Fig. 6: Frequency dependence graphs for the final variant of the equivalent circuit based on the Laplasian operator and the semi-segment series RC equivalent circuit (gray graph) indicated within the "Rmax" and "Rmin" limits for the porcine muscle

IV. EXPERIMENTS RESULTS

Thus, we obtained a simple mathematic expression describing frequency dependence of the biological tissue impedance with the high accuracy degree complying with the results of the real impedance measuring scanning relative to the homogeneous fragment.

By that the final variant of the biological tissue equivalent circuit got a very compact and multipurpose structure. In case of replacing such different tissue fragments as the muscle tissue and liver varieties are observed in three constants only.

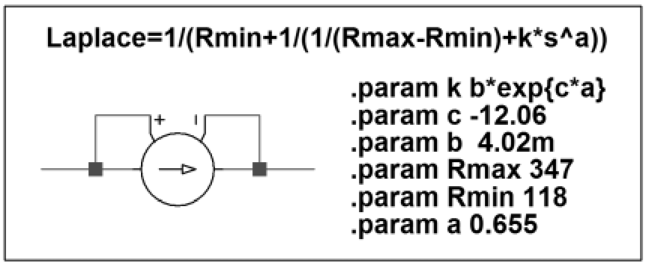

Fig. 7: Final variant of the equivalent circuit based on the Laplasian operator and adjusting parameters of the biological tissue sample for the porcine muscle tissue

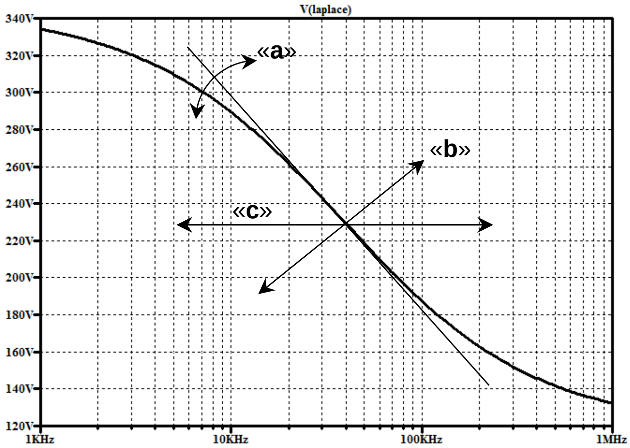

Fig. 8: The frequency dependence graph for the final equivalent circuit based on the Laplasian operator for the porcine muscle tissue simulating with indication of the adjusting factors effect direction

These are the impedance values on the frequency range limits represented by *"Rmax"* and *"Rmin"* resistanse, as well as the *"b"* adjusting constant that displays biological tissue specificity, and probably correlated with the polarization factor typical for this certain types of tissue.

For the sake of convenient formation and adjustment of the equivalent circuit variants, below are showed direction of the adjusting constants effect on the form of the resulting graph for the entire circuit, through the example of the frequency dependence graph for the porcine muscle tissue.

The «*a*» power index changes characteristics slope, the «*c*» exponential constant shifts the graph in parallel to the X axis, and the «*b*» constant shifts it close to the orthogonal direction at the point of graph crossing the mean value line representing the limit voltages.

The adjusting constants indicated above act relatively well at small deviations from the set values. In certain cases, the larger changes can cause some nonlinearities.

However, the practice is the truth criterion, and thus, it is always possible to determine degree of applicability of the obtained model in given certain application by comparing results of the simulation with those of the measurements.

V. RESULTS DISCUSSION

Obtained results of computer simulation showed a high degree of their compliance with the measuring data in both, the value and the form of the impedance frequency function curve.

The developed variant of the equivalent electrical circuit can be used as a suitable base for the joint simulation of medical devices used electrical contact with biological tissue.

VI. SUMMARY AND CONCLUSIONS

Application of the fractional derivatives in equivalent electrical circuits through the biological tissue example showed their high efficiency and relative simplicity of application for achieving results of computer simulation close to the real objects parameters.

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